Parallel algorithm for computing the characteristic polynomials of polynomial matrices[1]

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Abstract. There is produced a parallel algorithm for computing the characteristic polynomials of polynomial matrices. The algorithm is based on the method of homomorphic images in the ring of integers and in the ring of polynomials. We discuss the implementation of algorithms in the system Mathpar.

Let k be the number of processors in cluster. Zero processor is called a root processor.

Let $A \in Z^{n\times n}[x_1, \ldots, x_t]$; M be a list of prime numbers and each processor has the list M.

The parallel algorithm consists of four steps.

Step 1. Computation the number of modules.

The root processor computes the size of the integer coefficients and the largest degrees of the variables $x_1, \ldots, x_t$ of the characteristic polynomial. These estimations were obtained in work [1]. The root processor receives the quantity of polynomial modules and the quantity numerical modules. We choose the numerical modules from the list M and the polynomial modules from the set $x_i,\ x_i-1,\ x_i-2, \ldots,$ $i = 1, \ldots, t$.

The root processor sends the matrix A, the quantity of polynomial modules and the quantity numerical modules to all processors. As a result each processor has the input matrix A, the quantity of polynomial modules and the quantity numerical modules.

Step 2. Solution of the problem over finite fields.

If we have m_i polynomial modules $x_i, x_i - 1, \ldots, x_i - (m_i - 1)$, $i = 1, \ldots, t$ and s numerical modules then necessary to solve $q = sm_1m_2 \cdots m_t$ problems over finite fields.

Denote by j any remainder of q by k. Then i-th processor computes quantity r_i of problems over finite fields where r_i is equal $\left\lfloor \frac{q}{k} \right\rfloor + 1$ for $i = 0, \ldots, j-1$ or

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r_i is equal $\left\lfloor \frac{q}{k} \right\rfloor$ for $i = j, \ldots, k-1$. Consequently, the task is distributed to all processors proportionally.

We used an algorithm for computing characteristic polynomials over a finite field from work [2, 3].

As a result each processor has an array of polynomials in a new variable with coefficients over a finite field.

Step 3. Recovery coefficients of a characteristic polynomial over ring $Z[x_1, \ldots, x_t]$.

Each processor recovers one polynomial by the r polynomials obtained in step 2.

Coefficients of the polynomial over ring $Z[x_1, \ldots, x_t]$ are recovered by means of Newton's algorithm [4].

The root processor divides each polynomial into k parts. The root processor sends this division to all processors.

Each processor divides polynomials into k parts. What is more, all j-th parts of polynomials consist of monomials of the same degree, $j = 0, \ldots, k-1$. And for $i \neq j$ j-th part and i-th part don't contain monomials of the same degree.

Processors send parts to each other, so that the j-th processor receives the j-th parts, $j = 0, \ldots, k-1$.

Each processor recovers one polynomial by k polynomials.

As a result each processor has a part of the characteristic polynomial.

Step 4. Result gathering at the root processor.

Each processor sends its polynomial to the root processor. The root processor adds the received polynomials.

As a result we have a characteristic polynomial of the input matrix at the root processor.

The considered algorithm for computing the characteristic polynomials of polynomial matrices is was realized in the computer algebra system Mathpar [5]-[7].

We note this algorithm equips with proportional distribution of tasks on the cluster nodes and provides good scalability.

References

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