# The Laplace method for linear differential equations with impulse coefficients[1]

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**Abstract.** There are considered linear differential equations with impulse coefficients and composite right hand parts. Solving with the Laplace method is produced.

We call traditionally by an impulse function a function $h(t) = k_i, t \in [t_{i-1}, t_i),\ i = 1, \dots, m+1,\ t_{m+1} = \infty$. It may be written by means of the Heaviside function $H$: $h(t) = \sum_{i=1}^{m} k_i \left(H(t - t_{i-1}) - H(t - t_i)\right) + k_{i+1} H(t - t_i)$.

Consider an equation

$$\sum_{n=1}^{N} \left( \sum_{i=1}^{m} k_i \left(H(t - t_{i-1}) - H(t - t_i)\right) + k_{m+1} H(t - t_m \right) y^{(n)}(t) = f(t), \qquad (1)$$

where $f(t)$ is a composite function, which by means of the Heaviside function, may be written as follows

$$f(t) = \sum_{i=1}^{m} f_i(t) \left(H(t - t_{i-1}) - H(t - t_i)\right) + f_{m+1} H(t - t_m).$$

As input data we take values of the unknown function and its derivatives up to the order $n-1$: $y(t_i), y^{(k)}(t_i),\ i = 1, \dots, m,\ k = 1, \dots, N$.

As a result of the Laplace transform we obtain the algebraic equation:

$$\sum_{n=1}^{N} \left(K_0^n + K^n p^n Y(p)\right) = F(p), \qquad (2)$$

where

$$K_0^n = k_1 e^{-pt_0} \sum_{k=2}^{n} y^{(k-1)}(t_0) p^{k-2} + \sum_{i=1}^{m} (k_{i+1} - k_i) e^{-pt_i} \sum_{k=2}^{n} y^{(k-1)}(t_i) p^{k-2} +$$

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$$k_{m+1}e^{-pt_m}\sum_{k=2}^{n}y^{(k-1)}(t_m)p^{k-2},$$

$$K^n = k_1e^{-pt_0} + \sum_{i=1}^{m}(k_{i+1}-k_i)e^{-pt_i} + k_{m+1}e^{-pt_m},$$

and $F(p)$ is the Laplace image of $f(t)$ ([1]).

Solving the algebraic equation (2) and performing the inverse Laplace transform, we obtain the solution of the equation (1).

Similarly linear systems of differential equations may be treated.

## References

[1] Malaschonok N.A. Solving differential equations by parallel Laplace method with assured accuracy. Serdica, Journal of Computing, Sofia, Bulgaria, 2007. Vol. 33, No. 4, 20-35.

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