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# Calculations on a Cluster with Distributed Memory: Matrix Decomposition and Inversion in the Commutative Domain

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J. Dongarra at his talk at International Congress ICMS-2016 [1] put attansion on the several difficult challenges. He noted that the task of managing calculations on a cluster with distributed memory for algorithms with sparse matrices is today one of these the most difficult challenges.

Here we must also add problems with the type of the basic algebra: matrices can be over fields or over commutative rings. For sparse matrices, it is not true that all computations over polynomials or integers can be reduced to computations in finite fields. Such reduction may be not effective for sparse matrices.

We consider the class of block-recursive matrix algorithms. The most famous of them are standard and Strassen's block matrix multiplication, Schur-Strassen's block-matrix inversion [2].

Block-recursive algorithms were not so important as long as the calculations were performed on computers with shared memory. The generalization of Strassen's matrix inversion algorithm [2] with additional permutations of rows and columns by J. Bunch and J. Hopkroft [3] is not a block-recursive algorithm. Only in the nineties it became clear that block-recursive matrix algorithms are required to operate with sparse super large matrices on a supercomputer with distributed memory.

The block recursive algorithm for the solution of systems of linear equations and for adjoint matrix computation which is some generalisation of Schur-Strassen inversion in commutative domains was discraibed in [7], [8] and [10]. See also at the book [9]. However, in all these algorithms, except matrix multiplication, a very strong restriction are imposed on the matrix. The leading minors, which are on the main diagonal, should not be zero.

This restriction was removed later. The algorithm that computes the adjoint matrix, the echelon form, and the kernel of the matrix operator for the commutative domains was proposed in [11]. The block-recursive algorithm for the Bruhat decomposition and the LDU decomposition for the matrix over the field was obtained in [12], and these algorithms were generaized for the matrices over commutative domains in [14] and in [15].

Here is a brief history of development of the matrix recursive agorithms in integral domain.

Algorithms for solution of a system of linear equations of size $n$ in an integral domain, which served as the basis for creating recursive algorithms:
(1983) Forward and backward algorithm ($\sim n^3$) [4].
(1989) One pass algorithm ($\sim \frac{2}{3}n^3$) [5].
(1995) Combined algoritm ($\sim \frac{7}{12}n^3$ for corner block size $r = \frac{n}{2}$) [6].

Recursive algorithms for solution of a system of linear equations and for adjoint matrix computation in an integral domain without permutations:
(1997) Recursive algorithm for a linear equations system solution [7].
(2000) Adjoint matrix computation (with 6 levels) [8].
(2006) Adjoint matrix computation (with 5 levels) [10].

Main recursive algorithms for sparse matrices:
(2008) Computation of kernel, adjoint and inverse matrices [11].
(2010) Bruhat and LEU decompositions in the feilds [12].
(2012) Bruhat and LDU decompositions in the domains [13], [14].

New achivements:
(2013) It is proved that the LEU algorithm has the complexity $O(n^2 r^{\beta-2})$ for rank $r$ matrices [19].
(2015) Bruhat and LDU decompositions in the domains (alternative algorithm) [15].
(2017) It is proved that the LEU algorithm has the complexity $O(n^2 s^{\beta-2})$ for quasiseparable matrix, if any it's submatrix which entirely below or above the main diagonal has small rank $s$ [20].

The block-recursive matrix algorithms for sparse matrix require a special approachs to managing parallel programs. One approach to the cluster computations management is a scheme with one dispatcher (or one master).

We consider another scheme of cluster menagement. It is a scheme with multidispatching, when each involved computing module has its own dispatch thread and several processing threads [21], [22].

We demonstrate the results of experiments with parallel programms on the base of multidispatching.

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